Theoretische Physik IV: Statistische Mechanik und Thermodynamik

Problem Set No. 8

Due on: Friday, 20.6.08 in the practice groups

Exercise 8.1 *(Magnetization of a paramagnet)* **(10 points)**

The energy E of N spins $\mathbf{s}_n$ in a magnetic field $\mathbf{H}$ is given by

$$E = -\mu \sum_{n=1}^{N} \mathbf{H} \cdot \mathbf{s}_n$$

. Calculate the free energy $F = -k_B T \ln Z$ and the magnetization $M = -\left.\frac{\partial F}{\partial H}\right|_T$

(a) for *Isingspins*, i.e. $s_n = \pm 1$ with only two orientations with respect to the magnetic field, either parallel or antiparallel w.r.t. H. (4 points)

(b) for *Heisenbergspins*, i.e. $\mathbf{s}_n = (s_x, s_y, s_z)_n$ are three-dimensional vectors of length $\| \mathbf{s}_n \| = 1$. (6 points)

Exercise 8.2 *(Potts-Model)* **(10 points)**

In this problem we investigate an often used and quite general model system: the Potts-Model. Consider N particles arranged in a row. We identify particle 1 with particle $N+1$. Each particle can take one out of p different states, which are denoted by $\nu = 1, \ldots, p$. The interaction energy of adjacent particles is $-J < 0$, if both particles are in the same state, 0 otherwise. The Hamiltonian of the system is given by

$$H(\nu_1, \ldots, \nu_N) = -J \sum_{j=1}^{N} \delta_{\nu_j, \nu_{j+1}}$$

(a) Determine the canonical partition sum of the system by means of the transfer matrix. (6 points)

(b) Calculate the internal energy $U(T)$ per particle in the thermodynamic limit $N \to \infty$. (3 points)

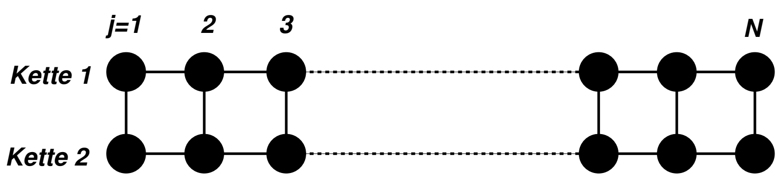
(c) Discuss the behaviour of the system both for high and low temperatures. (1 point)

Exercise 8.3 *(Ising-Band)* **(10 points)**

Consider two one-dimensional, ferromagnetic closed Ising chains of length N, which are arranged parallel and coupled (see figure). The Hamiltonian of the system is

$$H(\mathbf{s}_1, \ldots, \mathbf{s}_N) = -J_1 \sum_{j=1}^{N} (s_{j,1} s_{j+1,1} + s_{j,2} s_{j+1,2}) - J_2 \sum_{j=1}^{N} s_{j,1} s_{j,2}, \qquad J_1 > 0, J_2 > 0$$

Here, $\mathbf{s}_j = (s_{j,1}, s_{j,2}) \in \{+1, -1\}^2$ denotes the two-dimensional vector of the jth spins of the first and second chain. Let $\mathbf{s}_{N+1} = \mathbf{s}_1$.



(a) Show that the canonical partition sum can be written as

$$Z_N = \operatorname{Tr} T^N$$

where $T \in \mathbb{R}^{4\times 4}$ is an appropriate transfer matrix of the system. (8 points)

(b) Calculate the free energy F per particle in the thermodynamic limit $N \to \infty$ (2 points)

Hint: It might be useful to use a computer algebra program for determination of the eigenvalues of T!.