THEORETISCHE PHYSIK IV: STATISTISCHE MECHANIK UND THERMODYNAMIK

Problem Set No. 6

Due on: Friday, 6.6.08 in the practice groups

Exercise 6.1 *(Ideal Gas)* **(10 points)**

On heating a room let the air pressure remain constant (and equal to the pressure of the outer air). For a room with a volume of $100\,\mathrm{m}^3$, calculate the change in the internal energy and the entropy of the air inside the room, upon heating the air from $0°C$ to $20°C$.
Hint: Consider the air as an ideal gas and use the extensitivity of the entropy. The entropy/mol of air at 1 atm and $0°C$ is $S = 196\ \mathrm{J/(mol\ K)}$.

Exercise 6.2 *(Equation of State for a Photon Gas)* **(10 points)**

The classical ideal gas satisfies the well-known equation of state $pV = NkT$. For a gas of non-interacting photons however we have $pV = E/3$. Although we cannot derive this equation at this point of the lecture, we can derive the most important properties of this gas from the equation of state.

(a) Assume that the energy density $u = E/V$ only depends on temperature. Show that for the photon gas the Stefan-Boltzmann law $u = \sigma_0 T^4$ (σ_0=const.) holds. (5 points)
Hint: Consider $\frac{\partial E}{\partial V}\big|_T$ and use the Maxwell relation between $\frac{\partial S}{\partial V}\big|_T$ and $\frac{\partial P}{\partial T}\big|_V$ to obtain a differential equation for $u(T)$.

(b) Calculate the entropy of the photon gas. What is the exponent in the adiabatic equation $pV^\gamma = \text{const.}$? (5 points)

Exercise 6.3 *(Van-der-Waals Gas)* **(10 points)**

In the lecture the van-der-Waals' equation of state was derived, which can be written in the following form

$$p(V) = \frac{NkT}{V - Nb} - a\frac{N^2}{V^2} \tag{1}$$

(a) Determine the critical point (p_c, V_c) and the critical temperature T_c from the condition $p'(V) = p''(V) = 0$. (3 points)

(b) On writing the equation of state in the dimensionless variables $\tau = T/T_c$, $\nu = V/V_c$ and $\pi = p/p_c$ one can eliminate the constants a and b. First express a and b by T_c, V_c and p_c and then rewrite the van-der-Waals equation of state in the new variables τ, ν and π. (3 points)

(c) Use the equation of state (1) to calculate the free energy F, the entropy S and the internal energy U of the van-der-Waals gas. (The results for the ideal gas can be assumed to be known.) (4 points)