Theoretische Physik IV: Statistische Mechanik und Thermodynamik

10. Übungsblatt

Abgabedatum: Friday, 4.7.08 in den Übungen

Aufgabe 10.1 *(Bose gas)* **(10 Punkte)**

The grandcanonical partition sum of an ideal gas consisting of bosons can be written as (compare with the expression derived in the lecture)

$$\ln Z_{grk} = -\sum_\nu \ln\left(1 - e^{-\beta(E_\nu - \mu)}\right) \tag{1}$$

(a) Calculate

$$\langle N \rangle = \sum_\nu \langle n_\nu \rangle = \frac{1}{\beta}\frac{\partial}{\partial\mu} \ln Z_{grk}$$

and from this derive the mean occupation numbers $\langle n_\nu \rangle$ of the states. (3 Punkte)

(b) Calculate the entropy

$$S = k\left(1 - \beta\frac{\partial}{\partial\beta}\right) \ln Z_{grk}$$

of the Bose system. Derive an expression for S which only contains the mean occupation numbers $\langle n_\nu \rangle$. (5 Punkte)

(c) What is the entropy S in the limit of a classical system $\langle n_\nu \rangle \ll 1$? (2 Punkte)

Aufgabe 10.2 *(Ideal gas in the classical limit)* **(10 Punkte)**

(a) Calculate $\ln(Z_{grk})$ (cf. eq. (1)) for the Bose gas in a cube with edge length L in the thermodynamic limit for $\mu < 0$ and $e^{-\beta(E_\nu - \mu)} \ll 1$. For this purpose replace the sum over the states by an integral and expand the logarithm. Compare with the classical results of problem 5.1. (6 Punkte)

(b) Use the result of (a) to calculate the average particle number $\langle N \rangle$ and the energy $\langle E \rangle$. Express $\langle E \rangle$ by $\langle N \rangle$ and T. (4 Punkte)

Aufgabe 10.3 *(Quantum statistics of ideal gas systems)* **(10 Punkte)**

Here we want to have a look at the mean occupation number of states of quantum mechanical ideal gas system on a more general footing. Consider an ideal gas, whose atoms can occupy distinct energy levels E_ν. The occupation number of each energy level is limited to values $n_\nu \in \{0, 1, \ldots, l\}$.

(a) Calculate the grandcanonical partition sum of this system. (4 Punkte)

(b) Show that the mean occupation number for this system is given by

$$\langle n_\nu \rangle = \frac{1}{e^{\beta(E_\nu - \mu)} - 1} - \frac{l+1}{e^{\beta(E_\nu - \mu)(l+1)} - 1}$$

(3 Punkte)

(c) Calculate $\langle n_\nu \rangle$ for the limiting cases $l \to \infty$ and $l = 1$. $l \to \infty$ is the Bose gas known from the lecture, $l = 1$ is the Fermi gas. (3 Punkte)